

Please complete all problems as thoroughly as possible and remember to show your work.

1	12	
2	6	
3	12	
4	6	
5	8	
6	8	
7	8	
8	14	
9	10	
10	5	
11	6	
12	5	
E.C.	(6)	
Total	100	

Part I: Estimating limits numerically.

Use the tables of the values of  $g(x)$  for particular values of  $x$  to help answer the questions.

1.

$x$	1.9	1.99	1.999	2	2.001	2.01	2.1	7	25	100	1000	10000
$g(x)$	1.756	1.751	1.75001	5	1.74998	1.743	1.741	1	-10	-324	-74359	-46952053

Estimate the following limits as best as you can from the table.

i.  $(3) \lim_{x \rightarrow 2^-} g(x)$

ii.  $(3) \lim_{x \rightarrow 2^+} g(x)$

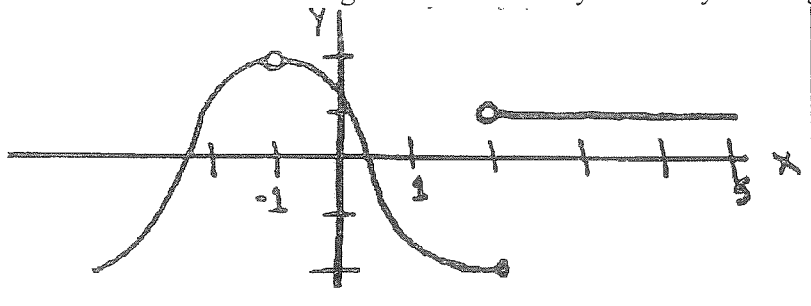
iii.  $(3) \lim_{x \rightarrow 2} g(x)$

iv.  $(3) \lim_{x \rightarrow \infty} g(x)$

2. (6) As far as you can determine from the table, where is the function  $g(x)$  discontinuous? For each discontinuity you find, explain which condition of continuity is broken and determine if the discontinuity is removable or non-removable.

Part II: Estimating limits graphically.

3. Determine the following limits as best as you can by looking at the graph of  $f(x)$  below.



i.  $(3) \lim_{x \rightarrow 2^+} f(x)$

ii.  $(3) \lim_{x \rightarrow 2^-} f(x)$

iii.  $(3) \lim_{x \rightarrow 4} f(x)$

iv.  $(3) \lim_{x \rightarrow 1} f(x)$

4. (6) As far as you can determine from the graph, where is the function  $f(x)$  discontinuous? For each discontinuity you find, explain which condition of continuity is broken and determine if the discontinuity is removable or non-removable.

Part III: Computing limits algebraically

For this part, COMPUTE the limits using algebraic techniques.

5. (8)  $\lim_{x \rightarrow \pi} \cos x$

6. (8)  $\lim_{x \rightarrow 3} \frac{x^2 + x - 2}{x - 3}$

7. (8)  $\lim_{x \rightarrow 2} \frac{x^2 + x - 2}{x^2 + 3x + 2}$

8. If  $f(x) = \begin{cases} 3x + 4 & x < -1 \\ 3 & x = -1 \\ x^2 & x > -1 \end{cases}$  sketch a graph of the function  $f(x)$  (2). Then find the limits below.

Show ALL work, even if you can compute the limits in your head. Without work you will not receive full credit.

ii. (6) Compute  $\lim_{x \rightarrow -5} f(x)$ .

iii. (6) Compute  $\lim_{x \rightarrow 1} f(x)$ .

9. (10) Find values of  $a$  and  $b$  that make the following function continuous.

$$f(x) = \begin{cases} 3x + a & x < 0 \\ e^x & 0 \leq x \leq \pi \\ b \cos x & x > \pi \end{cases}$$

10. (5) For a function  $f(x)$ , the expression  $\frac{f(x+h) - f(x)}{h}$  is called a difference quotient. If

$f(x) = \sqrt{x+1}$ , then the difference quotient, which we'll consider as a new function,  $g(x)$ , is

$g(x) = \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$ . Limits of difference quotients will become extremely important in

Chapter 2.

In order to COMPUTE  $\lim_{h \rightarrow 0} g(x)$ , what strategy would you use? (I don't want you to have to do all of the algebra, but I want you to demonstrate to me you could take this limit if need be. A good way to explain your strategy might be to do the first step or two of computing the limit.)

Part III: Other

11. (6) If  $f(1)=2$  and  $f(2)=-4$ , what does the Intermediate Value Theorem say about the values of  $f$  on the interval  $[1,2]$ ? Explain your answer in complete sentences.

12. (5) Find the vertical and horizontal asymptotes of  $f(x) = \frac{3x+2}{2x-1}$ . Determine the behavior of  $f(x)$  on either side of the asymptote.

Extra Credit:

i. If  $\lim_{x \rightarrow 2} f(x) = 4$ , what can you conclude about  $f(2)$ ?

ii.  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ . Is this true or false? Explain your answer.



